

**NAME:**

## **Math 150 Exam 2**

**Instructions:** WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. If  $F(x) = f(g(x))$ , where  $f(-2) = 8$ ,  $f'(-2) = 4$ ,  $f'(5) = 3$ ,  $g(5) = -2$ , and  $g'(5) = 6$ , find  $F'(5)$ .

[10 pts]

2. Use chain rule to find the derivative of  $y = \left(\frac{x^2 + 1}{x^2 - 1}\right)^3$  [10 pts]

3. Let  $y(x)$  be given implicitly by the equation  $e^{x/y} = x - y$ . Find  $\frac{dy}{dx}$   
[10 pts]

4. Find the derivative for the function  $y = x^{\sin x}$ . [Hint: Use logarithmic differentiation]  
[10 pts]

5. A sample of tritium-3 decayed to 94.5% of its original amount after a year.
- (i) What is the half-life of tritium-3? [6 pts]
  - (j) How long would it take the sample to decay to 20% of its original amount? [4 pts]

6. A street light is mounted at the top of a 15-ft tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole? [10 pts]

7. Use linear approximation to estimate the value of  $e^{-0.015}$  [10 pts]

8. Show that  $\sqrt{1+x} < 1 + \frac{1}{2}x$  for all  $x > 0$  [10 pts]

9. Calculate  $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1-4x}}{x}$  [10 pts]

10. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is a maximum? How should the wire be cut so that the area is minimal? [10 pts]

**Extra-Credit**

11. Establish the derivative formula for the function  $y = \sin^{-1} x$  by using implicit differentiation. [10 pts]

12. Find a function  $f$ , whose  $n$ th derivative at  $x = 0$  is  $f^{(n)}(0) = 5^n n!$ . [10 pts]

13. State and prove the Mean-Value-Theorem. [10 pts]

14. Suppose  $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ . Show that for every integer  $p$ ,  $f(p) = [f(1)]^p$ . [10 pts]